information about binding energies and to make isotopic assignments of resonances. The sum-coincidence spectra yield the results 6.16±0.05 and 5.44±0.09 MeV for the binding energies of W^{182} (21.2 eV) and W^{186} (18.8 eV), respectively; these values are not in disagreement with the values 6.29±0.04 and ≈4.8 MeV obtained in other ways.[40] The maximum energy of the sum pulses observed in the spectrum for the 15.9-eV resonance is 6.30±0.08 MeV. This energy implies that the resonance must be assigned to either W^{180} or W^{182}. Similarly, a previously reported weak resonance at 75 eV must be assigned to either W^{180} or W^{182}.

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Atomic Masses from Gallium to Molybdenum*

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The 16-in. double-focusing mass spectrometer at the University of Minnesota has been employed to measure the atomic mass of 42 stable isotopes in the region $A=69$ to 100. The standard error associated with these results is approximately 5 parts in 10^8. Improvements in the instrument are described that result in an increase of useful resolution of a factor of 2 to 3. A set of 64 radioactive masses is calculated from the stable mass data together with β-decay energies and nuclear reaction Q values. The resultant table of masses is used to calculate total nuclear binding energies, separation energies and pairing energies for a number of nuclei in the region near $N=50$. The systematics of the separation energies display very smooth characteristics except at the shell closure. Neutron pairing energies show a marked decrease in value following the shell closure.

INTRODUCTION

THE 16-in. double-focusing mass spectrometer at the University of Minnesota has been employed in the past to measure atomic masses in several regions of the periodic table. Measurements of atomic masses for most of the stable isotopes with $A<70$ have been reported.[1–5] In addition, krypton and xenon,[6] lead and mercury[7] masses have been measured. Operational difficulties in the mass spectrometer become progressively more apparent in measurements of heavy isotopes where maximum resolution is required. These difficulties necessitated the movement and reconstruction of the instrument.

The improved instrument has been employed to measure a number of stable masses for $A>70$. Mass doublets in the region from gallium through molybdenum are reported in this paper and the following paper[8] reports results in the region from ruthenium through xenon.

The mass results have been employed to study the nuclear binding energy systematics in the region $Z=31$ to 42 and $N=36$ to 58. These data include the $N=50$ shell closure and also the possible $N=40$ and $Z=40$ subshell closure.

THE INSTRUMENT

The instrument employed for all previous measurements has been described in some detail elsewhere.[9] The ion optics of the instrument are arranged to yield first- and second-order angle focusing and first-order velocity focusing at the fixed collector slit. Doublets are measured by the peak-matching method.

One of the recurring difficulties in the original instrument was the random modulation of the ion beam resulting from building vibrations and time-varying magnetic fields from nearby ac power lines. The modulation not only limited the maximum useful resolution of the mass spectrometer, but also limited the sweep frequency that could be employed.

To remove these difficulties, the instrument was rebuilt in a ground-floor room which had lower stray magnetic fields. The instrument was mounted on a 2-ton cast-iron surface plate. The use of the surface plate as an

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accurately plane reference surface greatly aided in the precise location of the various elements of the spectrometer. The spectrometer and surface plate are supported on four spring mounts, the resonant frequency of the supported mass being about 2 cps. This system then effectively isolates the spectrometer from the higher building vibration frequencies. In order to further minimize the effects of vibration, the spectrometer housing is mounted rigidly to the surface plate. Focusing adjustments are now made by moving the magnet on a track system. The modification also included redesign of several of the adjustable slits so as to give more precise control of the ion beam.

The changes made in the instrument have resulted in improved performance. The maximum usable resolution of the improved instrument is 2 or 3 times that of the original spectrometer. Other changes have considerably decreased the time necessary for focusing the instrument.

MEASUREMENTS

The mass spectrometer has the property that the mass of the ion collected is proportional to the resistance of a circuit element which determines the electric field in the instrument. Thus, one can show that the equation which relates doublet width to resistance is

$$\Delta M = M\Delta R/R, \qquad (1)$$

where ΔM is the doublet width, M is the mass of one member of the doublet which corresponds to resistance value R, and ΔR is the change in resistance necessary to cause the second ion group to be collected. By measuring known doublets, Eq. (1) may be shown to be correct to a high degree of accuracy.

In the past, this instrument was employed mainly to measure narrow doublets made up of a hydrocarbon comparison ion of known mass and an ion of unknown mass at the same mass number. For the mass region under consideration in this paper, these doublets have $\Delta M/M$ values of approximately 10^{-3}. It has been shown that the instrument is capable of measuring wider doublets with sufficient accuracy to be worthwhile.[6] One type of wide doublet is the doublet composed of two adjacent isotopes of the same element. Doublets of this sort are known as isotopic doublets and have been employed previously to supplement the hydrocarbon doublet data.

One of the difficulties that arises when hydrocarbon comparison ions are employed is caused by the rare isotope of carbon, C^{13}. When the hydrocarbon $C_m{}^{12}H_n{}^1$ is used as a comparison ion, a satellite ion is also present, $C^{13}C_{m-1}{}^{12}H_{n-1}{}^1$. In the mass region considered in this paper, a resolution of about 1/50 000 is required to completely resolve the C^{13} satellite. Resolution of this instrument is defined as $\Delta M/M$ where ΔM corresponds to the width at half-height of an ion peak of mass M. The resolution values ranged from 1/60 000 to 1/200 000 during this period. Thus, the C^{13} satellite was at all times completely resolved.

TABLE I. Mass doublets.

Doublet[a]	Mass difference[b] (mu)	Error[c]
$C_5H_9-Ga^{69}$	144.852 7	24
$C_5H_{11}-Ga^{71}$	161.370 2	32
$C_5H_{10}-Ge^{70}$	154.001 3	22
$C_4H_8O-Ge^{70}H_2$	117.616 1	18
$C_4H_8O-Ge^{72}$	135.438 4	21
$C_4H_9O-Ge^{73}$	141.878 4	21
$CS_2-Ge^{74}H_2$	7.314 0	14
CS_2-Ge^{76}	22.741 6	15
$Ge^{70}H_2-Ge^{72}$	17.821 3	17
$Ge^{72}H-Ge^{73}$	6.443 9	13
$Ge^{73}H_2-Ge^{74}H$	10.105 1	17
$Ge^{74}H_2-Ge^{76}$	15.425 0	17
$C_3H_7O_2-As^{75}$	123.009 8	26
$C_6H_2-Se^{74}$	93.173 8	38
$C_6H_4-Se^{76}$	112.099 9	81
$C_6H_5-Se^{77}$	119.211 9	42
$C_6H_6-Se^{78}$	129.642 6	22
$C_6H_8-Se^{80}$	146.068 5	29
$C_6H_{10}-Se^{82}$	161.545 0	46
$C_6H_8-HBr^{79}$	136.444 3	24
$C_6H_{10}-HBr^{81}$	154.135 3	38
$Br^{81}-HBr^{79}$	990.135 1	125
$C_6H_6-Kr^{78}$	126.584 3[d]	36
$C_6H_8-Kr^{80}$	146.225 7[d]	46
$C_6H_{10}-Kr^{82}$	164.769 8[d]	34
$C_6H_{11}-Kr^{83}$	171.946 8[d]	34
$C_6H_{12}-Kr^{84}$	182.399 4[d]	25
$C_6H_{14}-Kr^{86}$	198.936 7[d]	27
$Kr^{83}-Kr^{82}$	1000.647 9[d]	120
$Kr^{84}-Kr^{83}$	997.371 6[d]	120
$C_6H_{13}-Rb^{85}$	189.927 6	39
$C_4H_7O_2-Rb^{87}$	135.417 8	27
$C_6H_{12}-Sr^{84}$	180.470 8	26
$C_6H_{14}-Sr^{86}$	200.264 9	36
$C_4H_7O_2-Sr^{87}$	135.722 2	35
$C_4H_8O_2-Sr^{88}$	146.789 1	47
$Sr^{87}-Sr^{86}$	999.618 1	115
$Sr^{88}-Sr^{87}$	996.739 6	116
$C_7H_5-Y^{89}$	133.247 0	34
$C_4H_{10}O_2-Zr^{90}$	163.377 1	55
$C_7H_7-Zr^{91}$	149.143 1	44
$C_7H_8-Zr^{92}$	157.569 4	38
$C_7H_{10}-Zr^{94}$	171.929 4	39
$C_7H_{12}-Zr^{96}$	185.628 0	57
$Zr^{91}-Zr^{90}$	1000.942 0	116
$Zr^{92}-Zr^{91}$	999.397 2	117
$C_7H_9-Nb^{93}$	164.046 9	35
$C_7H_8-Mo^{92}$	155.790 0	32
$C_7H_{10}-Mo^{94}$	173.159 6	32
$C_7H_{11}-Mo^{95}$	180.236 5	35
$C_7H_{12}-Mo^{96}$	189.226 9	30
$C_5H_5O_2-Mo^{97}$	122.937 6	23
$C_5H_6O_2-Mo^{98}$	131.375 4	28
$C_7H_{16}-Mo^{100}$	217.730 3	42
$Mo^{95}-Mo^{94}$	1000.757 2	122
$Mo^{96}-Mo^{95}$	998.838 5	124
$Mo^{97}-Mo^{96}$	1001.346 3	123
$Mo^{98}-Mo^{97}$	999.386 0	121

[a] Throughout this paper C, H, O, and S refer to C^{12}, H^1, O^{16}, and S^{32}, respectively.

[b] Mass differences are given in milliunits. All masses and mass differences in this work are measured in a scale in which the atomic mass of C^{12} is exactly equal to 12 units (symbol u). The symbol mu refers to one milliunit on the unified mass scale. The symbol μu refers to one microunit.

[c] Throughout this paper the errors refer to the last significant figure of the particular result. The errors given in this table are taken from the original experimental data. The resulting error in an atomic mass calculation will be rounded off to the nearest μu.

[d] For completeness, the krypton doublets have been included. They are taken from the M.S. thesis of R. R. Ries, University of Minnesota, 1959 (unpublished).

Metal ions for these measurements were obtained from a variety of sources. In two cases, gases were employed. These were hydrogen bromide and germanium tetrahydride. Pure metals were used for gallium, arsenic, and strontium. In the remaining cases, the following metallic compounds were employed: selenium dioxide, rubidium chloride, yttrium chloride, zirconium tetrachloride, niobium pentachloride, and molybdenum trioxide. Metals and metallic compounds were heated when necessary to obtain sufficient vapor pressure. Sample heating took place in a source furnace essentially the same as that previously described.[3] Two baffles have been added between the sample and the ionization region to improve the vapor distribution.

RESULTS

The experimental mass doublet differences are presented in Table I. Both the narrow hydrocarbon-isotope doublets and the wide isotopic doublets are included in this table. The mass scale employed is the unified mass scale adopted by the International Union of Pure and Applied Physics in 1960. In this scale, the atomic mass of C^{12} is exactly 12 units (symbol u). The errors listed are standard errors and refer to the last quoted figure. These errors are calculated from resistor uncertainties and the standard error of the mean of the original data. The details of the resistor error assignments may be found in an article by Quisenberry *et al.*[4]

The doublet values of Table I are combined with the atomic masses of certain secondary mass standards in order to calculate atomic masses. Table II lists the atomic masses of the stable atoms in the mass region $69 \leqq A \leqq 100$. The secondary mass standards which were employed to calculate atomic masses from the doublet data are recorded in Table III. In some cases, the mass of a particular atom was overdeterminded by measurement of isotopic doublets in addition to the usual narrow hydrocarbon-isotope doublets. In these cases, a weighted least-squares adjustment was carried out on the data, and the best values found in this process are recorded as the results in Table II. The error associated with an atomic mass is the square root of the sum of the square of the error on the appropriate doublet value and the square of the error in the hydrocarbon reference mass. Whenever the least-squares adjustment was possible, the error on these atomic masses is just the error resulting from the least-squares calculation.

The final column of Table II provides a direct comparison between the present and other previous mass spectroscopic values. For comparison purposes, mass values were calculated from previous doublet results using the standard masses listed in Table III. The first half of this column contains the former Minnesota values determined by Collins *et al.*[10] on a smaller, less precise mass spectrometer. These values have errors approximately 10 times larger than those of the present investigation. No systematic difference seems to be evident for the 28 atomic mass comparisons which can be made, since 14 of the former results are higher and

TABLE II. Atomic masses computed from the doublet data of Table I together with a comparison with previous mass spectroscopic values.

Isotope	Present result[a] u	Error	1961 Mass table[b] u	Error	Other results[c] u	Error
Ga^{69}	68.925 569	3	68.925 682	28	68.925 72[d]	4
Ga^{71}	70.924 701	4	70.924 840	50	70.924 82[d]	8
Ge^{70}	69.924 247	2	69.924 277	20	69.924 00[d]	6
Ge^{72}	71.922 075[e]	2	71.921 740	50	71.921 60[d]	5
Ge^{73}	72.923 457	2	72.923 360	70	72.923 34[d]	3
Ge^{74}	73.921 177	2	73.921 150	60	73.921 00[d]	6
Ge^{76}	75.921 402	2	75.921 360	90	75.921 28[d]	4
As^{75}	74.921 591	3	74.921 580	50	74.921 72[d]	4
Se^{74}	73.922 476	4	73.922 450	60	73.922 54[d]	7
Se^{76}	75.919 199	8	75.919 229	48	75.919 27[d]	4
Se^{77}	76.919 912	4	76.919 934	48		
Se^{78}	77.917 305	2	77.917 348	48		
Se^{80}	79.916 529	3	79.916 512	17	79.916 47[d]	4
Se^{82}	81.916 702	5	81.916 660	70	81.916 64[d]	4
Br^{79}	78.918 328	3	78.918 348	19	78.918 40[d]	5
Br^{81}	80.916 287	4	80.916 344	37	80.916 42[d]	5
Kr^{78}	77.920 364	4	77.920 368	5	77.920 19[d]	8
Kr^{80}	79.916 372	5	79.916 388	13		
Kr^{82}	81.913 477	4	81.913 483	8	81.913 46[d]	6
Kr^{83}	82.914 125	4	82.914 131	8	82.914 06[d]	5
Kr^{84}	83.911 497	3	83.911 504	5	83.911 51[d]	5
Kr^{86}	85.910 609	4	85.910 617	8	85.910 80[d]	6
Rb^{85}	84.911 793	4	84.911 710	60	84.912 03[d]	6
Rb^{87}	86.909 183	3	86.909 180	80	86.909 31[d]	17
Sr^{84}	83.913 425	3	83.913 376	11	83.913 399[f]	17
					83.913 25[d]	15
Sr^{86}	85.909 278	4	85.909 260	80	85.909 156[f]	23
					85.909 36[d]	10
					85.909 357[g]	50
Sr^{87}	86.908 882	4	86.908 890	80	86.908 816[f]	42
					86.908 99[d]	6
Sr^{88}	87.905 634	5	87.905 610	90	87.905 485[f]	16
					87.905 678[g]	18
					87.906 01[d]	11
Y^{89}	88.905 876	4	88.905 430	90	88.905 862[f]	15
					88.905 72[d]	11
Zr^{90}	89.904 696	5	89.904 320	90	89.904 672[f]	20
					89.904 931[g]	8
					89.904 33[d]	25
Zr^{91}	90.905 631	4	90.905 250	100	90.905 629[f]	20
Zr^{92}	91.905 028	4	91.904 590	110	91.905 093[f]	22
Zr^{94}	93.906 317	4	93.906 140	360	93.906 268[f]	25
Zr^{96}	95.908 268	6	95.908 200	800	95.908 379[f]	46
Nb^{93}	92.906 375	4	92.906 020	110	92.906 315[f]	25
					92.905 66[d]	8
Mo^{92}	91.906 807	4	91.906 290	130	91.906 869[f]	42
Mo^{94}	93.905 086	4	93.904 740	130	93.905 166[f]	52
Mo^{95}	94.905 835	4	94.905 720	360	94.905 841[f]	15
Mo^{96}	95.904 670	3	95.904 550	360	95.904 685[f]	44
Mo^{97}	96.906 014	3	96.905 750	400	96.905 952[f]	30
Mo^{98}	97.905 401	3	97.905 510	410	97.905 425[f]	16
Mo^{100}	99.907 464	5	99.907 570	490	99.907 543[f]	35

[a] The atomic masses are computed from the doublet values of Table I and the values of the secondary mass standards listed in Table III.
[b] See Ref. 12.
[c] In the original references, these results were presented in the old O^{16} scale. They have been changed to comply to the new C^{12} mass scale.
[d] See Ref. 10.
[e] This result is the weighted average of the two narrow Ge^{72} doublets listed in Table I.
[f] See Ref. 11.
[g] N. R. Isenor, R. C. Barber, and H. E. Duckworth, Can. J. Phys. 38, 819 (1960).

[10] T. L. Collins, W. H. Johnson, and A. O. Nier, Phys. Rev. 94, 398 (1954).

14 lower than the present work. Because of the comparatively large error of the previous results, the comparison of results is not particularly valuable as a test of the reliability of the present data.

Starting with strontium at $A=84$, a comparison is made with the work carried out by Demirkhanov *et al.*[11] These results have errors that are about 5 times those quoted for the present results. A comparison of the 14 stable isotopic masses from Y^{89} to Mo^{100} reveals moderate agreement between the present results and the results of Demirkhanov. This order of agreement does not hold in the case of several of the strontium masses (Sr^{86} and Sr^{88}) where the disagreement between the two results to more than 5 times their combined errors. Smaller disagreements occur at Zr^{92}, Zr^{96}, and Nb^{93}. The reason for these discrepancies is unknown.

The present results are also compared with the values found in the Mass Table due to König *et al.*[12] The results of König represent "best" values from a least-squares adjustment of much of the available nuclear reaction and disintegration data, together with selected mass spectroscopic information. It should be mentioned that in the mass region from $A=69$ to $A=93$, the mass results of Collins *et al.*[10] served as the mass spectroscopic input data for the least-squares adjustment of the Mass Table. The comparison of the present results with this table is quite good from $A=73$ to $A=88$. A particularly encouraging feature of this table is the fact that, with three exceptions, the table always adjusted the older Minnesota results of Collins in the right direction to agree with the more precise, present results. Until recently, precise mass information from $A=88$ to $A=100$ was not available. This is reflected in the poor agreement of the Mass Table results with the present work in this mass region. The discrepancies are particularly obvious for the zirconium and molybdenum isotopes, where the mass table values also have very large errors.

TABLE III. Secondary mass standards.

Standard	Present value u	Error	Reference
C^{12}	12.000 000 0		a
H^1	1.007 824 7	2	b
n[c]	1.008 665 4	4	d
H^2	2.014 102 2	1	d
N^{14}	14.003 073 1	4	e
O^{16}	15.994 914 2	5	b
S^{32}	31.972 073 8	11	d
Cl^{35}	34.968 853 1	19	f

[a] This is the definition of the C^{12} mass scale.
[b] See Ref. 5.
[c] The symbol n stands for the neutron mass.
[d] See Ref. 12.
[e] See Ref. 2.
[f] See Ref. 3.

TABLE IV. Least-squares adjusted values for overdetermined stable isotopes and isotopic differences.

Isotope or isotopic difference	Measured result[a] u	Error	Final adjusted[b] result u	Error	Difference[c] μu
Ge^{70}	69.924 246	2	69.924 247	2	+1
Ge^{72}	71.922 073	3	71.922 075	2	+2
Ge^{73}	72.923 458	3	72.923 457	2	−1
Ge^{74}	73.921 180	3	73.921 177	2	−3
Ge^{76}	75.921 402	3	75.921 402	2	0
$Ge^{72}-Ge^{70}$	1.997 828	2	1.997 828	2	0
$Ge^{73}-Ge^{72}$	1.001 381	2	1.001 382	2	+1
$Ge^{74}-Ge^{73}$	0.997 720	2	0.997 720	2	0
$Ge^{76}-Ge^{74}$	2.000 224	2	2.000 224	3	0
Br^{79}	78.918 328	3	78.918 328	3	0
Br^{81}	80.916 287	4	80.916 287	4	0
$Br^{81}-Br^{79}$	1.997 960	12	1.997 959	5	−1
Kr^{82}	81.913 477	4	81.913 477	4	0
Kr^{83}	82.914 125	4	82.914 125	4	0
Kr^{84}	83.911 497	3	83.911 497	3	0
$Kr^{83}-Kr^{82}$	1.000 648	12	1.000 648	5	0
$Kr^{84}-Kr^{83}$	0.997 372	12	0.997 372	5	0
Sr^{86}	85.909 280	4	85.909 278	4	−2
Sr^{87}	86.908 879	4	86.908 882	4	+3
Sr^{88}	87.905 637	5	87.905 634	5	−3
$Sr^{87}-Sr^{86}$	0.999 618	12	0.999 603	5	−15
$Sr^{88}-Sr^{87}$	0.996 740	12	0.996 752	6	+12
Zr^{90}	89.904 698	6	89.904 696	5	−2
Zr^{91}	90.905 630	5	90.905 631	4	+1
Zr^{92}	91.905 028	4	91.905 028	4	0
$Zr^{91}-Zr^{90}$	1.000 942	12	1.000 935	7	−7
$Zr^{92}-Zr^{91}$	0.999 397	12	0.999 397	6	0
Mo^{94}	93.905 087	4	93.905 086	4	−1
Mo^{95}	94.905 835	4	94.905 835	4	0
Mo^{96}	95.904 669	4	95.904 670	3	+1
Mo^{97}	96.906 014	3	96.906 014	3	0
Mo^{98}	97.905 401	3	97.905 401	3	0
$Mo^{95}-Mo^{94}$	1.000 757	12	1.000 749	5	−8
$Mo^{96}-Mo^{95}$	0.998 838	12	0.998 834	5	−4
$Mo^{97}-Mo^{96}$	1.001 346	12	1.001 345	4	−1
$Mo^{98}-Mo^{97}$	0.999 386	12	0.999 387	4	+1

[a] Calculated from the doublet data of Table I and the standard masses listed in Table III. These results serve as input data for the least-squares adjustment.
[b] These results and errors are the least-squares adjusted values.
[c] Adjusted data minus originally measured results (in microunits).

Hydrocarbon mass unit doublets of the form $C_mH_n-C_mH_{n-1}$ were measured concurrently with the doublets of Table I in order to make a continuous check on the validity of the dispersion relation, Eq. (1). The average of these mass unit measurements, taken during the measurement of any one element, was used to calculate a value for the hydrogen atomic mass from Eq. (1); and this result was compared with the accepted value for hydrogen listed in Table III. The ratio of the accepted value of the hydrogen mass to the measured value of the hydrogen mass, as calculated from the mass unit doublet, is called the dispersion constant. Thus, when the mass unit measurements lead to results which agree on the average with the accepted mass of hydrogen, the dispersion constant is just equal to unity, and Eq. (1) is used as it stands. However, any deviation from the accepted value of hydrogen defines a dispersion constant different from unity. All doublets which are measured concurrently with this particular mass unit

[11] R. A. Demirkhanov, V. V. Dorokhov, and M. I. Dzkuya, Zh. Eksperim. i Teor. Fiz. **40**, 1572 (1961) [translation: Soviet Phys.—JETP **13**, 1104 (1961)].
[12] L. A. König, J. H. E. Mattauch, and A. H. Wapstra, Nucl. Phys. **31**, 18 (1962).

TABLE V. Comparison of mass spectrometer and nuclear reaction mass differences for stable isotopes.

Mass difference	Present result u	Error	Nuclear reaction result[a] u	Error	Reaction employed	Reference[b]
$Ge^{73}-Ge^{72}$	1.001 382	2	1.001 687	172	$Ge^{73}(\gamma,n)Ge^{72}$	c
$Se^{77}-Se^{76}$	1.000 713	9	1.000 703	10	$Se^{76}(n,\gamma)Se^{77}$	d
$Se^{78}-Se^{77}$	0.997 394	5	0.997 411	15	$Se^{77}(n,\gamma)Se^{78}$	d
$Kr^{84}-Kr^{83}$	0.997 372	5	0.997 317	21	$Kr^{83}(n,\gamma)Kr^{84}$	5-2-10, 6
$Rb^{87}-Sr^{87}$	0.000 301	5	0.000 293	3	$Rb^{87}(\beta^-)Sr^{87}$	60-3-61
			0.000 261	32	$Rb^{87}(p,n)Sr^{87}$	e
$Sr^{87}-Sr^{86}$	0.999 603	5	0.999 629	19	$Sr^{86}(n,\gamma)Sr^{87}$	d
			0.999 540	215	$Sr^{86}(d,p)Sr^{87}$	60-3-62
$Sr^{88}-Sr^{87}$	0.996 752	6	0.996 737	15	$Sr^{87}(n,\gamma)Sr^{88}$	d
$Zr^{91}-Zr^{90}$	1.000 935	7	1.000 888	32	$Zr^{90}(d,p)Zr^{91}$	f
$Zr^{92}-Zr^{91}$	0.999 397	6	0.999 332	32	$Zr^{91}(d,p)Zr^{92}$	g
			0.999 368	43	$Zr^{91}(n,\gamma)Zr^{92}$	d
$Mo^{96}-Mo^{95}$	0.998 834	5	0.998 831	11	$Mo^{95}(n,\gamma)Mo^{96}$	d
$Mo^{97}-Mo^{96}$	1.001 345	4	1.001 436	322	$Mo^{96}(d,p)Mo^{97}$	h
			1.001 043	322	$Mo^{97}(\gamma,n)Mo^{96}$	60-6-49
$Mo^{98}-Mo^{97}$	0.999 387	4	0.999 772	107	$Mo^{97}(d,p)Mo^{98}$	h

[a] These results were calculated from the appropriate Q value and the masses listed in Table III.
[b] The series of numbers in this column refer to the page numbers of the *Nuclear Data Sheets* of Ref. 16.
[c] See Ref. 17.
[d] See Ref. 14.
[e] A. J. Elwyn, H. H. Landon, S. Oleksa, and G. N. Glasoe, Phys. Rev. **112**, 1200 (1958).
[f] R. L. Preston, H. J. Martin, Jr., and M. B. Sampson, Phys. Rev. **121**, 1741 (1961).
[g] H. J. Martin, Jr., M. B. Sampson, and R. L. Preston, Phys. Rev. **125**, 942 (1962).
[h] N. S. Wall, Phys. Rev. **96**, 664 (1954).

doublet are then corrected by the appropriate amount. Dispersion constants were determined for each element during this investigation and they ranged from 0.999 967±2 to 1.000 015±3. The average value of this constant for the entire block of measurements was 0.999 997±2.

The effect of the least-squares adjustment on the data where isotopic masses were overdetermined can be seen in Table IV. The second column lists the results which are calculated directly from the doublet data and which then serve as the input data for the adjustment. The third column contains the results of the least-squares process, while the last column records the difference between the adjusted value and the input data. It can be seen that the atomic masses change at most by 2 or 3 microunits, and these changes are always covered by the original errors. The adjusted errors on the atomic mass values generally remain the same or become one microunit smaller. The isotopic mass differences are generally adjusted by a larger amount, as in the case of strontium where the two isotopic mass differences are changed by 12 and 15 microunits. With the exception of the $Sr^{87}-Sr^{86}$ doublet, these adjustments are also covered by the original errors.

A comparison of mass differences determined both from mass spectroscopic measurements and from nuclear reaction energies is contained in Table V. This table records mass differences only when nuclear reactions connecting 2 stable atoms are available. The conversion factor between Q values and mass units is taken from Cohen[13] and is given by 1 unit = 931.476 ±0.004 MeV.

The number of comparisons in this mass region which involve only 2 stable atoms is not very large, but the over-all agreement is quite good. The agreement is particularly good for recent precise nuclear reaction results. Particular mention should be made of the excellent agreement between the present results and the (n,γ) reaction results determined by Kinsey and Bartholomew.[14] With the exception of the $Sr^{87}-Sr^{86}$ mass difference, all of these results agree with the present work within the combined errors. This comparison also verifies the gamma spectrum assignments made by Kinsey and Bartholemew when determining the ground-state Q values.

Nuclear reaction and disintegration energies are combined with the stable mass data of Table II in order to calculate the atomic masses of radioactive atoms. The results of such calculations are recorded in Table VI, which lists a final adopted mass for 65 radioactive atoms in the mass region from $A=67$ to $A=100$. A schematic view of this mass region, exhibiting the various paths by which the radioactive masses may be calculated from the stable atomic masses, is presented in Fig. 1. Several zinc masses from the 1961 Mass Table[11] were utilized to calculate some radioactive gallium masses.

As is evident from Fig. 1, some radioactive masses may be calculated from the stable masses in only one way, while others may be determined by several reaction paths. When only one such path is available, the final adopted value is just the calculated result, so this entry is included only in the final column. In the 25 cases where the radioactive mass can be calculated in more than one way, the result of each calculation is recorded in the second column. The final adopted value for such a radioactive mass is then the weighted average of the several individual results.

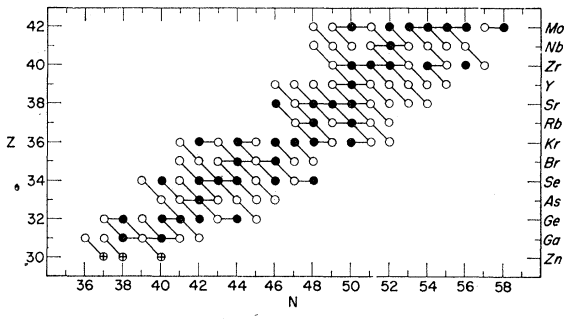

FIG. 1. Reaction scheme showing the stable nuclei whose atomic masses have been measured in this investigation. Solid circles represent stable nuclei, open circles represent radioactive nuclei, and connecting lines indicate available nuclear reaction and β-decay mass differences. The three crossed circles represent zinc isotopes not measured in this work.

[13] E. R. Cohen, Bull. Am. Phys. Soc. **7**, 305 (1962).
[14] B. B. Kinsey and G. A. Bartholomew, Can. J. Phys. **31**, 1051 (1953).

TABLE VI. Atomic masses of radioactive isotopes computed from measured isotopic masses of Table II and available nuclear reaction and beta-decay energies.

Isotope	Calculated mass[a] u	Error	Nuclear reaction employed	Reference[b]	Final adopted value[c] u	Error
Ga^{67}			$Zn^{67}(p,n)Ga^{67}$	d	66.928 220	12
Ga^{68}			$Zn^{68}(p,n)Ga^{68}$	d	67.927 996	11
Ga^{70}	69.925 936	22	$Ga^{69}(n,\gamma)Ga^{70}$	d		
	69.926 018	11	$Ga^{70}(\beta^-)Ge^{70}$	d		
	69.926 048	17	$Zn^{70}(p,n)Ga^{70}$	d	69.926 012	20
	69.925 956	65	$Ga^{71}(\gamma,n)Ga^{70}$	e		
Ga^{72}	71.926 359	11	$Ga^{72}(\beta^-)Ge^{72}$	e		
	71.926 372	9	$Ga^{71}(n,\gamma)Ga^{72}$	f	71.926 367	7
	71.925 883[g]	19	$Ga^{71}(n,\gamma)Ga^{72}$	e		
Ga^{73}			$Ga^{73}(\beta^-)Ge^{73}$	59-1-37	72.925 123	43
Ge^{69}	68.927 956	5	$Ga^{69}(p,n)Ge^{69}$	h		
	68.927 971	13	$Ge^{69}(\beta^+)Ga^{69}$	d	68.927 958	5
	68.928 571[g]	215	$Ge^{70}(\gamma,n)Ge^{69}$	i		
Ge^{71}	70.924 966	32	$Ga^{71}(p,n)Ge^{71}$	59-2-44		
	70.924 951	5	$Ge^{71}(\epsilon)Ga^{71}$	59-2-43	70.924 951	5
Ge^{75}	74.922 858	22	$Ge^{75}(\beta^-)As^{75}$	e	74.922 857	22
	74.922 720	215	$Ge^{76}(\gamma,n)Ge^{75}$	e		
Ge^{77}			$Ge^{77}(\beta^-)As^{77}$	59-3-44	76.923 598	55
As^{73}			$Ge^{73}(p,n)As^{73}$	e	72.923 855	32
As^{74}	73.923 936	11	$As^{74}(\beta^-)Se^{74}$	59-4-71		
	73.923 938	31	$As^{75}(\gamma,n)As^{74}$	e	73.923 932	8
	73.923 928	11	$As^{74}(\beta^+)Ge^{74}$	e		
As^{76}	75.922 387	13	$As^{76}(\beta^-)Se^{76}$	59-5-39	75.922 390	12
	75.922 420	43	$As^{75}(n,\gamma)As^{76}$	59-5-41		
As^{77}			$As^{77}(\beta^-)Se^{77}$	59-3-46	76.920 646	11
As^{78}			$As^{78}(\beta^-)Se^{78}$	59-5-48, 50	77.921 889	215
As^{79}			$As^{79}(\beta^-)Se^{79}$	59-2-49	78.920 969	107
Se^{73}			$Se^{73}(\beta^+)As^{73}$	59-1-41	72.926 807	34
Se^{75}			$As^{75}(p,n)Se^{75}$	59-1-48	74.922 520	4
Se^{79}			$Se^{79}(\beta^-)Br^{79}$	59-2-50	78.918 500	6
Se^{81}			$Se^{81}(\beta^-)Br^{81}$	59-1-64	80.917 790	54
Br^{76}			$Br^{76}(\beta^+)Se^{76}$	59-5-43	75.924 127	13
Br^{77}			$Se^{77}(p,n)Br^{77}$	59-3-49	76.921 376	6
Br^{78}	77.921 156	11	$Se^{78}(p,n)Br^{78}$	59-5-52	77.921 156	11
	77.921 096	150	$Br^{79}(\gamma,n)Br^{78}$	e		
Br^{80}	79.918 519	12	$Br^{80}(\beta^-)Kr^{80}$	59-1-56		
	79.918 556	7	$Br^{80}(\beta^+)Se^{80}$	59-1-56	79.918 544	10
	79.918 496	38	$Br^{81}(\gamma,n)Br^{80}$	e		
	79.918 535	14	$Br^{79}(n,\gamma)Br^{80}$	e		
Br^{82}			$Br^{82}(\beta^-)Kr^{82}$	59-1-70	81.916 796	4
Br^{83}			$Br^{83}(\beta^-)Kr^{83}$	59-1-80	82.915 199	22
Kr^{77}			$Kr^{77}(\beta^+)Br^{77}$	e	76.924 468	22
Kr^{79}	78.920 221	54	$Kr^{78}(d,p)Kr^{79}$	59-2-53	78.920 068	14
	78.920 067	6	$Kr^{79}(\beta^+)Br^{79}$	59-2-52		
Kr^{81}			$Kr^{80}(d,p)Kr^{81}$	58-1-39	80.916 605	107
Kr^{85}	84.912 514	9	$Kr^{85}(\beta^-)Rb^{85}$	60-3-40	84.912 516	6
	84.912 519	9	$Kr^{84}(d,p)Kr^{85}$	j		
Kr^{87}	86.913 499	215	$Kr^{87}(\beta^-)Rb^{87}$	60-3-60	86.913 363	52
	86.913 354	54	$Kr^{86}(d,p)Kr^{87}$	j		
Kr^{88}			$Kr^{88}(\beta^-)Rb^{88}$	60-3-70	87.914 266	235
Rb^{84}	83.914 392	14	$Rb^{84}(\beta^-)Sr^{84}$	5-2-11		
	83.914 363	11	$Rb^{84}(\beta^+)Kr^{84}$	5-2-11	83.914 374	10
	83.914 411	86	$Rb^{85}(\gamma,n)Rb^{84}$	k		
	83.914 561[g]	86	$Rb^{85}(\gamma,n)Rb^{84}$	l		
Rb^{86}	85.911 186	7	$Rb^{86}(\beta^-)Sr^{86}$	60-3-50	85.911 186	7
	85.911 173	43	$Rb^{87}(\gamma,n)Rb^{86}$	60-3-61		
Rb^{88}	87.911 217	107	$Rb^{88}(\beta^-)Sr^{88}$	60-3-71	87.911 260	96
	87.911 435	215	$Rb^{87}(d,p)Rb^{88}$	60-3-71		
Rb^{89}			$Rb^{89}(\beta^-)Sr^{89}$	60-3-82	88.911 654	54
Sr^{85}	84.912 959	322	$Sr^{86}(\gamma,n)Sr^{85}$	60-3-52	84.912 981	33
	84.912 981	33	$Rb^{85}(p,n)Sr^{85}$	60-3-42		
Sr^{89}	88.907 447	6	$Sr^{89}(\beta^-)Y^{89}$	60-3-83	88.907 446	9
	88.907 339	75	$Sr^{88}(d,p)Sr^{89}$	60-3-83		
Sr^{90}			$Sr^{90}(\beta^-)Y^{90}$	60-4-33	89.907 755	32
Sr^{91}			$Sr^{91}(\beta^-)Y^{91}$	60-5-74	90.910 150	16
Sr^{92}			$Sr^{92}(\beta^-)Y^{92}$	60-5-84	91.910 949	81
Y^{85}			$Y^{85}(\beta^+)Sr^{85}$	j	84.916 728	37
Y^{86}			$Y^{86}(\beta^+)Sr^{86}$	j	85.914 968	54
Y^{87}			$Sr^{87}(p,n)Y^{87}$	60-3-63	86.910 693	215
Y^{88}	87.909 526	12	$Y^{88}(\beta^+)Sr^{88}$	m		
	87.909 329[g]	33	$Y^{88}(\beta^+)Sr^{88}$	n		
	87.909 654	86	$Y^{89}(\gamma,n)Y^{88}$	l	87.909 522	8
	87.909 901[g]	54	$Y^{89}(\gamma,n)Y^{88}$	o		
	87.909 519	8	$Sr^{88}(p,n)Y^{88}$	p		
Y^{90}	89.907 133	12	$Y^{90}(\beta^-)Zr^{90}$	60-4-34		
	89.907 189	10	$Y^{89}(n,\gamma)Y^{90}$	q	89.907 171	32
	89.907 420	54	$Y^{89}(d,p)Y^{90}$	r		
Y^{91}			$Y^{91}(\beta^-)Zr^{91}$	60-5-75	90.907 284	12
Y^{92}			$Y^{92}(\beta^-)Zr^{92}$	60-5-85	91.908 893	32
Y^{93}			$Y^{93}(\beta^-)Zr^{93}$	60-5-98	92.909 547	24
Y^{94}			$Y^{94}(\beta^-)Zr^{94}$	60-5-11	93.911 685	215
Zr^{89}	88.908 923	8	$Zr^{89}(\beta^+)Y^{89}$	60-3-85		
	88.908 678	97	$Zr^{90}(\gamma,n)Zr^{89}$	60-4-36	88.908 923	10
	88.908 923	8	$Y^{89}(p,n)Zr^{89}$	60-3-85		
Zr^{93}	92.906 443	4	$Zr^{93}(\beta^-)Nb^{93}$	60-5-99	92.906 444	10
	92.906 550	43	$Zr^{92}(d,p)Zr^{93}$	60-5-99		
Zr^{95}	94.908 097	54	$Zr^{94}(d,p)Zr^{95}$	60-5-127	94.908 037	13
	94.908 034	12	$Zr^{95}(\beta^-)Nb^{95}$	60-5-127		
Zr^{97}			$Zr^{97}(\beta^-)Nb^{97}$	60-6-47	96.910 944	33
Nb^{89}			$Nb^{89}(\beta^+)Zr^{89}$	60-3-86	88.913 088	98
Nb^{90}			$Nb^{90}(\beta^+)Zr^{90}$	60-4-37	89.911 255	12
Nb^{91}			$Mo^{91}(\beta^+)Nb^{91}$	60-5-78	90.906 795	82
Nb^{92}	91.906 871	215	$Zr^{92}(p,n)Nb^{92}$	e		
	91.907 186	41	$Nb^{93}(\gamma,n)Nb^{92}$	60-5-100	91.907 184	47
	91.907 325	161	$Nb^{92}(\beta^+)Zr^{92}$	60-5-81		
Nb^{94}	93.907 322	32	$Nb^{93}(n,\gamma)Nb^{94}$	60-5-114		
	93.907 253	107	$Nb^{93}(d,p)Nb^{94}$	60-5-114	93.907 314	27
	93.907 308	54	$Nb^{94}(\beta^-)Mo^{94}$	60-5-113		
Nb^{95}			$Nb^{95}(\beta^-)Mo^{95}$	60-5-128	94.906 832	6
Nb^{97}			$Nb^{97}(\beta^-)Mo^{97}$	60-6-48	96.908 088	8
Mo^{90}			$Mo^{90}(\beta^+)Nb^{90}$	60-4-40	89.913 982	108
Mo^{91}			$Mo^{92}(\gamma,n)Mo^{91}$	60-5-88, 81	90.911 583	75
Mo^{93}	92.906 898	43	$Nb^{93}(p,n)Mo^{93}$	60-5-101	92.906 953	70
	92.907 041	54	$Mo^{92}(d,p)Mo^{93}$	60-5-101		
Mo^{99}			$Mo^{100}(\gamma,n)Mo^{99}$	61-1-66	98.907 495	322

[a] For comparison purposes, this column lists the results of the calculations whenever more than one nuclear reaction was available for a particular isotope. No entry is made in this column when only one reaction energy is used. In these cases, the final adopted value is also the calculated value.

[b] Whenever possible, references are taken from large compilations of nuclear data. References to the original literature are made if that is the only source or if the experimental results lead to calculated masses which differ by large amounts. The sequence of numbers in this column refer to the year, the set, and the page numbers of the *Nuclear Data Sheets* of Ref. 16.

[c] The final adopted value is the weighted average of the various calculated results from different reactions. When only one reaction energy is available, the adopted value is simply the calculated result. The error associated with the adopted values is the larger of the internal error and the external error as defined in the *Nuclear Data Sheets*.

[d] F. Everling, L. A. König, J. H. E. Mattauch, and A. H. Wapstra, Nucl. Phys. **25**, 177 (1961).

[e] L. A. König, J. H. E. Mattauch, and A. H. Wapstra, Nucl. Phys. **28**, 1 (1961).

[f] See Ref. 15.

[g] This result is not included in the weighted average.

[h] See Ref. 12.

[i] D. M. Van Patter and Ward Whaling, Rev. Mod. Phys. **29**, 757 (1957).

[j] A. H. Wapstra (private communication).

[k] See Ref. 17.

[l] See Ref. 18.

[m] See Ref. 20.

[n] See Ref. 19.

[o] See Ref. 22.

[p] See Ref. 21.

[q] G. A. Bartholomew, P. J. Campion, J. W. Knowles, and G. Manning, Nucl. Phys. **10**, 590 (1959).

[r] N. S. Wall, Phys. Rev. **96**, 670 (1954).

In 15 of the 25 cases where several reactions were available to determine one radioactive mass, the results agree very well; see, for example, As^{74}. When the several independent reaction and disintegration energies combined with different stable atomic masses lead to these similar results, both the energy values and the mass

values are confirmed. In the remaining 10 cases, different reaction paths lead to results that do not agree within the combined error. Several of these cases will now be discussed individually.

The energies of the gamma rays in the complex spectrum resulting from the gallium (n,γ) reaction were measured by Campion *et al.*,[15] but the unambiguous assignment of the ground-state gamma energy of Ga^{72} was not possible. A listing of the unassigned gamma rays was given by these authors. An estimate of 7.1 MeV for the last neutron binding energy in Ga^{72} was given in the *Nuclear Data Sheets*.[16] This would suggest that the unassigned 6.971-MeV gamma corresponds to the ground-state reaction. However, this Q value leads to a Ga^{72} mass which differs by about 475 keV from the result calculated from the beta decay of Ga^{72} to Ge^{72}. The level schemes for these 2 isotopes seem to be well known and thus we have chosen the beta disintegration energy in order to calculate the mass of Ga^{72}. Assuming that result as correct, we have then assigned the 6.516-MeV gamma as the ground-state gamma for the $Ga^{71}(n,\gamma)Ga^{72}$ reaction.

The (γ,n) reactions for Rb^{84} are available. This reaction is difficult to interpret because of the large nuclear spin change involved. The value by Tobin *et al.*[17] determined by studying the reaction to the isomeric state of Rb^{84} is preferred to the value by Geller *et al.*[18] because of its agreement with the two well-known beta decays of Rb^{84}.

The positron decay energy of Y^{88} to Sr^{88} has been a controversial value for some time. Two distinct groups of results were found for this disintegration energy, and they are best represented by Ramaswamy *et al.*[19] who quotes the value 3442 ± 31 keV, and by Rhode *et al.*[20] who determined the value 3625 ± 10 keV. Two conflicting $Y^{89}(\gamma,n)Y^{88}$ reaction results were not previously helpful in resolving the discrepancy because of the large error in the mass values for Y^{89}. The recent and precise $Sr^{88}(p,n)Y^{88}$ reaction measured by Shafroth,[21] however, gives a result which agrees closely with the disintegration energy of Rhode. In addition, the present mass determination of Y^{89} permits a more precise calculation of Y^{88} from the two (γ,n) reactions. The mass result calculated from the Q value of Geller *et al.*[18] agrees reasonably well with the results due to Rhode and Shafroth. The (γ,n) result of Chidley *et al.*[22] is far outside the error of these values, and still further from the result of Ramaswamy's measurement. The final adopted value chosen here for the mass of Y^{88} is the weighted average of the three results due to Rhode, Shafroth, and Geller.

In the remaining cases where the agreement between the several possible calculations was not good, evidence did not seem to favor one value over another, so the final adopted value in these cases is just the weighted average.

NUCLEAR SYSTEMATICS

The atomic masses of stable and radioactive isotopes listed in Table II and Table IV may be employed to calculate the nuclear binding energies for various combinations of nucleons. In order to calculate the total nuclear binding energy (TNBE), the proper atomic mass value is substituted into Eq. (2)

$$\mathrm{TNBE}(Z,N)=ZM_{\mathrm{H}}+NM_n-{}_ZM_N{}^A-E_b(Z,N)/c^2, \quad (2)$$

where M_{H} is the hydrogen mass, M_n is the neutron mass, ${}_ZM_N{}^A$ is the atomic mass of the atom with Z protons and N neutrons. The term $E_b(Z,N)$ is an estimate of the total electronic binding energy. The functional form of this relation was taken from Foldy.[23] The value ranged from 51 μu for gallium to 104 μu for molybdenum, with a stated accuracy of 10%. Consideration of the small binding energy of the electron in hydrogen was neglected in view of the much larger uncertainty in the value of $E_b(Z,N)$.

Values of the total nuclear binding energy are useful in the calculation of various binding energy quantities of interest in the study of nuclear systematics. One quantity which indicates the general tendency of nuclear binding is the average binding energy, $\mathrm{TNBE}(Z,N)/A$. The binding energy of a given system of nucleons in a particular nucleus may be found by forming the appropriate difference between two total binding energy terms. As an example, the neutron separation energy and the proton separation energy are found from Eq. (3) and Eq. (4), respectively.

$$S_n(Z,N)=\mathrm{TNBE}(Z,N)-\mathrm{TNBE}(Z,N-1), \quad (3)$$

$$S_p(Z,N)=\mathrm{TNBE}(Z,N)-\mathrm{TNBE}(Z-1,N). \quad (4)$$

Table VII lists the total nuclear binding energy, the average nuclear binding energy, the neutron separation energy, and the proton separation energy in nuclei for which these quantities may be calculated from the present mass values.

In Fig. 2, the average nuclear binding energy for stable isotopes is plotted as a function of the mass number. The even-A points for each element are connected by a solid line and the odd-A points for all the elements are connected by a dashed line. The character-

[15] P. J. Campion and G. A. Bartholomew, Can. J. Phys. **35**, 1361 (1957).
[16] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington 25, D. C., 1958–1961).
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[19] M. H. Ramaswamy and P. S. Jastram, Nucl. Phys. **19**, 243 (1960).
[20] J. I. Rhode, O. E. Johnson, and W. G. Smith, Phys. Rev. **129**, 815 (1963).
[21] S. M. Shafroth, Nucl. Phys. **28**, 649 (1961).
[22] B. G. Chidley, L. Katz, and S. Kowalski, Can J. Phys. **36**, 407 (1958).
[23] L. L. Foldy, Phys. Rev. **83**, 397 (1951).

TABLE VII. Total nuclear binding energy, average binding energy per nucleon, and nucleon separation energies for those nuclei between $A=67$ and $A=100$ where sufficient data is available.

Isotope	Z	N	TNBE[a] mu	TNBE/A[b] mu	Error	S_n[c] mu	Error	S_p[d] mu	Error
Zn^{66}	30	36	620.600[e]						
Zn^{67}	30	37	628.165[e]						
Zn^{68}	30	38	639.114[e]						
Zn^{69}	30	39	645.992[e]						
Zn^{70}	30	40	655.962[e]						
Zn^{71}	30	41	662.005[e]						
Zn^{72}	30	42	670.901[e]						
Ga^{67}	31	36	626.249	9.3470	3			5.649	16
Ga^{68}	31	37	635.138	9.3403	3	8.889	16	6.973	16
Ga^{69}	31	38	646.230	9.3657	3	11.092	11	7.116	9
Ga^{70}	31	39	654.453	9.3493	4	8.223	20	8.461	35
Ga^{71}	31	40	664.430	9.3582	3	9.977	20	8.468	16
Ga^{72}	31	41	671.429	9.3254	3	6.999	8	9.424	220
Ga^{73}	31	42	681.338	9.3334	6	9.909	44	10.437	224
Ge^{69}	32	37	642.997	9.3188	3			7.859	12
Ge^{70}	32	38	655.374	9.3625	3	12.377	5	9.144	3
Ge^{71}	32	39	663.335	9.3427	3	7.961	5	8.882	21
Ge^{72}	32	40	674.876	9.3733	3	11.541	5	10.446	4
Ge^{73}	32	41	682.160	9.3447	3	7.284	3	10.731	7
Ge^{74}	32	42	693.105	9.3663	3	10.945	3	11.767	43
Ge^{75}	32	43	700.091	9.3345	4	6.986	22		
Ge^{76}	32	44	710.212	9.3449	3	10.121	22		
Ge^{77}	32	45	716.680	9.3075	8	6.468	55		
As^{73}	33	40	680.917	9.3276	5			6.041	32
As^{74}	33	41	689.506	9.3176	3	8.589	33	7.346	8
As^{75}	33	42	700.512	9.3402	3	11.006	9	7.407	3
As^{76}	33	43	708.378	9.3208	3	7.866	12	8.287	25
As^{77}	33	44	718.788	9.3349	3	10.410	16	8.576	11
As^{78}	33	45	726.210	9.3104	28	7.422	215	9.530	222
As^{79}	33	46	735.796	9.3139	14	9.586	240		
Se^{73}	34	39	677.120	9.2756	5				
Se^{74}	34	40	690.117	9.3259	3	12.997	34	9.200	32
Se^{75}	34	41	698.738	9.3165	3	8.621	6	9.232	9
Se^{76}	34	42	710.725	9.3516	3	11.987	9	10.213	9
Se^{77}	34	43	718.678	9.3335	3	7.953	9	10.300	13
Se^{78}	34	44	729.949	9.3583	3	11.271	5	11.161	11
Se^{79}	34	45	737.420	9.3344	3	7.471	6	11.210	215
Se^{80}	34	46	748.056	9.3507	3	10.636	7	12.260	107
Se^{81}	34	47	755.461	9.3267	7	7.405	54		
Se^{82}	34	48	765.214	9.3319	3	9.753	54		
Br^{76}	35	41	704.951	9.2757	3			6.213	14
Br^{77}	35	42	716.367	9.3035	3	11.416	14	5.642	10
Br^{78}	35	43	725.253	9.2981	3	8.886	13	6.575	12
Br^{79}	35	44	736.746	9.3259	3	11.493	11	6.797	3
Br^{80}	35	45	745.196	9.3150	3	8.450	10	7.776	12
Br^{81}	35	46	756.118	9.3348	3	10.922	11	8.062	5
Br^{82}	35	47	764.274	9.3204	3	8.156	6	8.813	54
Br^{83}	35	48	774.537	9.3318	4	10.263	22	9.323	23
Kr^{77}	36	41	712.431	9.2524	4			7.480	26
Kr^{78}	36	42	725.200	9.2974	3	12.769	22	8.833	7
Kr^{79}	36	43	734.161	9.2932	3	8.961	14	8.908	18
Kr^{80}	36	44	746.523	9.3315	3	12.362	15	9.777	5
Kr^{81}	36	45	754.955	9.3204	13	8.432	107	9.759	107
Kr^{82}	36	46	766.749	9.3506	3	11.794	107	10.631	5
Kr^{83}	36	47	774.766	9.3345	3	8.017	5	10.492	5
Kr^{84}	36	48	786.060	9.3579	3	11.294	4	11.523	22
Kr^{85}	36	49	793.706	9.3377	3	7.646	7		
Kr^{86}	36	50	804.278	9.3521	3	10.572	10		
Kr^{87}	36	51	810.190	9.3125	7	5.912	52		
Kr^{88}	36	52	817.952	9.2949	27	7.762	241		
Rb^{84}	37	47	782.337	9.3135	3			7.571	11
Rb^{85}	37	48	793.583	9.3363	3	11.246	11	7.523	5
Rb^{86}	37	49	802.856	9.3355	3	9.273	8	9.150	9
Rb^{87}	37	50	813.524	9.3509	3	10.668	8	9.246	4
Rb^{88}	37	51	820.112	9.3195	11	6.588	96	9.922	109
Rb^{89}	37	52	828.384	9.3077	7	8.272	110	10.432	241
Sr^{84}	38	46	782.440	9.3148	3				
Sr^{85}	38	47	791.549	9.3123	5	9.109	33	9.212	34
Sr^{86}	38	48	803.917	9.3479	3	12.368	33	10.334	5
Sr^{87}	38	49	812.979	9.3446	3	9.062	5	10.123	8
Sr^{88}	38	50	824.892	9.3738	3	11.913	6	11.368	5
Sr^{89}	38	51	831.746	9.3455	3	6.854	10	11.634	96
Sr^{90}	38	52	840.102	9.3345	4	8.356	33	11.718	63
Sr^{91}	38	53	846.373	9.3008	3	6.271	36		
Sr^{92}	38	54	854.239	9.2852	9	7.866	83		
Y^{85}	39	46	786.957	9.2583	5			4.517	37
Y^{86}	39	47	797.382	9.2719	7	10.425	65	5.833	63
Y^{87}	39	48	810.322	9.3140	25	12.940	220	6.405	215
Y^{88}	39	49	820.159	9.3200	3	9.837	215	7.180	9
Y^{89}	39	50	832.470	9.3536	3	12.311	9	7.578	6
Y^{90}	39	51	839.841	9.3316	4	7.371	32	8.095	33
Y^{91}	39	52	848.393	9.3230	3	8.552	34	8.291	34
Y^{92}	39	53	855.450	9.2984	4	7.057	34	9.077	36
Y^{93}	39	54	863.461	9.2845	4	8.011	40	9.222	84
Y^{94}	39	55	869.988	9.2552	23	6.527	216		
Zr^{89}	40	49	828.578	9.3099	3			8.419	13
Zr^{90}	40	50	841.470	9.3497	3	12.892	11	9.000	6
Zr^{91}	40	51	849.201	9.3319	3	7.731	7	9.360	32
Zr^{92}	40	52	858.469	9.3312	3	9.268	6	10.076	13
Zr^{93}	40	53	865.718	9.3088	3	7.249	11	10.268	34
Zr^{94}	40	54	874.510	9.3033	3	8.792	11	11.049	24
Zr^{95}	40	55	881.456	9.2785	3	6.946	14	11.468	215
Zr^{96}	40	56	889.890	9.2697	3	8.434	14		
Zr^{97}	40	57	895.880	9.2359	4	5.990	34		
Nb^{89}	41	48	823.566	9.2536	11				
Nb^{90}	41	49	834.064	9.2674	3	10.498	99	5.486	16
Nb^{91}	41	50	847.190	9.3098	9	13.126	83	5.720	82
Nb^{92}	41	51	855.466	9.2985	6	8.276	95	6.265	47
Nb^{93}	41	52	864.940	9.3004	3	9.474	47	6.471	5
Nb^{94}	41	53	872.667	9.2837	4	7.727	27	6.949	29
Nb^{95}	41	54	881.814	9.2823	3	9.147	28	7.304	7
Nb^{96}	41	55	889.260	9.2631	4	7.446	33	7.804	35
Nb^{97}	41	56	897.889	9.2566	3	8.629	33	7.999	10
Mo^{90}	42	48	830.491	9.2277	12			6.925	146
Mo^{91}	42	49	841.555	9.2479	9	11.064	131	7.491	76
Mo^{92}	42	50	854.996	9.2934	3	13.441	75	7.806	82
Mo^{93}	42	51	863.516	9.2851	8	8.520	70	8.050	84
Mo^{94}	42	52	874.048	9.2984	3	10.532	70	9.108	5
Mo^{95}	42	53	881.964	9.2838	3	7.916	5	9.297	27
Mo^{96}	42	54	891.796	9.2895	3	9.832	5	9.982	7
Mo^{97}	42	55	899.116	9.2692	3	7.320	4	9.856	32
Mo^{98}	42	56	908.395	9.2693	3	9.279	4	10.506	9
Mo^{99}	42	57	914.966	9.2421	33	6.571	320		
Mo^{100}	42	58	923.662	9.2366	3	8.696	320		

[a] Total nuclear binding energy in milliunits. The mass equivalent of the electronic binding energy has been subtracted from the total atomic binding energy. Errors are not listed because, for most purposes, the differences in 2 TNBE values are employed. An estimate of the error associated with TNBE may be found by multiplying the error assigned to TNBE/A by the corresponding A.

[b] Average nuclear binding energy per nucleon in milliunits.

[c] Neutron separation energy in milliunits.

[d] Proton separation energy in milliunits.

[e] The zinc masses were not measured in this investigation. In order to study the nuclear systematics of Ga and Ge it was necessary to calculate TNBE for the listed zinc isotopes. The atomic masses of zinc were taken from the 1961 Mass Table of Ref. 12.

istic parabolic shape in the even-A curves for each element appear in this region as has been observed in other regions. The obvious change in the curve connecting odd-A points near $A=89$ is a result of the shell closure at $N=50$. There is an increase in the odd-A curve in the region from $A=79$ to $A=89$. This rise does not appear at the shell closure at $N=82$[24] or $N=126$.[25]

[24] W. H. Johnson, Jr., and A. O. Nier, Phys. Rev. **105**, 1014 (1957).

[25] V. B. Bhanot, W. H. Johnson, and A. O. Nier, Phys. Rev. **120**, 235 (1960).

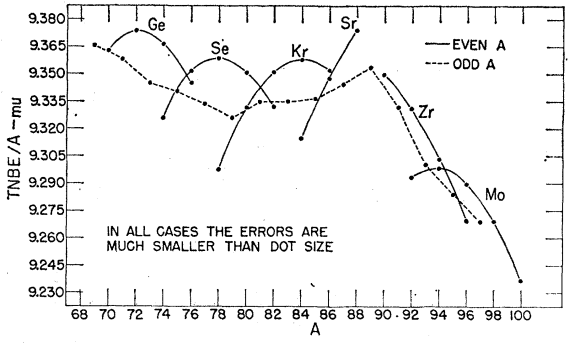

FIG. 2. Average binding energy per nucleon for stable isotopes.

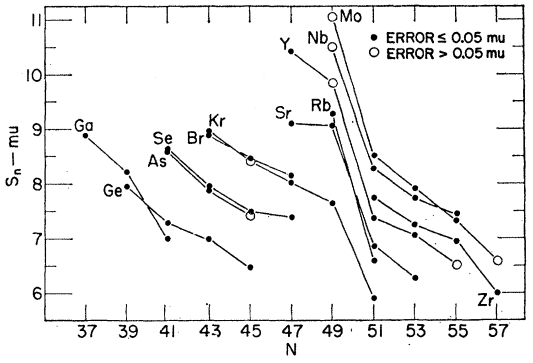

FIG. 4. Neutron separation energies for nuclei with an odd number of neutrons.

A more detailed study of binding-energy systematics may be made by considering the neutron separation energy and the proton separation energy. Figures 3 and 4 plot the neutron separation energy as a function of the neutron number for even N and odd N, respectively. In each case, successive points for a particular element are connected by a straight line. The sharp discontinuity beyond $N=50$ is shown in these graphs with greater precision than previously available. The generally smooth character of the curves on both sides of $N=50$ is perhaps the most significant result inferred from these plots. The $N=42$ data points for bromine and the $N=40$ data point for gallium seem to contradict this smooth behavior. In the case of bromine, the assignment of a mass to the radioactive Br^{77} and Br^{76} may be in error. In the case of gallium, these is no obvious error in either mass used to calculate this separation energy. This value may indicate an anomaly at $N=40$. The S_n values for germanium and selenium at $N=40$, however, show no particular anomaly.

Proton separation energies S_p have been plotted in Fig. 5 for even Z. A line connects data points of constant N. These curves indicate, as has been pointed out previously, that the closure of a neutron shell seems to have no effect on the proton separation energies. The variation of S_p for a given N value as a function of Z is also smooth. There is a persistent change in slope at $Z=40$ for the four curves that have data points at both $Z=38$ and $Z=42$. As in the neutron data, this may be an indication of a slight change in nuclear structure near nucleon number 40.

The study of the systematics of the binding energy of the last pair of nucleons in a nucleus is worthwhile because stable masses with small experimental errors are employed in most cases in the calculation. Table VIII lists the value of the binding energy of the last pair of neutrons S_{2n} for even N nuclei. These data are plotted in Fig. 6 as a function of neutron number N. Values from the same element are connected by a line. Once again the shell closure at $N=50$ is clearly visible. The smooth behavior of these curves for values on either side of $N=50$ is clearly evident.

The values of the binding energy of the last pair of protons for even Z nuclei are listed in Table IX. These data are plotted in Fig. 7. Once again the smooth variation of these data with changes in Z is evident. These results also indicate a change in slope at $Z=40$. Because there are only two other values of N for which there are 3 data points for a given N value, the consistency of slope for values of Z other than 40 is difficult to demonstrate.

A number of pairing energies for neutron and proton pairs may also be calculated from the mass data. The pairing energy for the last pair of neutrons added to a

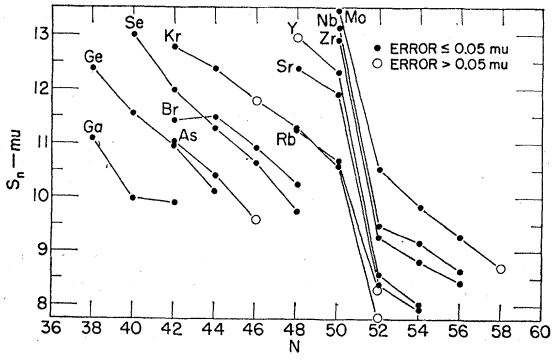

FIG. 3. Neutron separation energies for nuclei with an even number of neutrons.

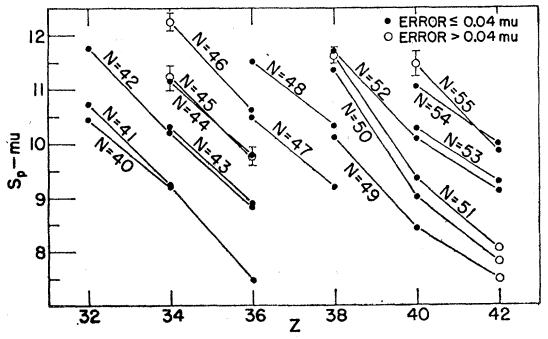

FIG. 5. Proton separation energies for nuclei with an even number of protons.

TABLE VIII. Binding energy and pairing energy of the last pair of neutrons for the listed isotopes.

Isotope	Z	N	S_{2n}[a] mu	Error	P_n[b] mu	Error
Ga^{69}	31	38	19.981	12	2.203	25
Ga^{71}	31	40	18.200	4	1.754	40
Ga^{73}	31	42	16.908	43	2.910	45
Ge^{72}	32	40	19.502	3	3.580	10
Ge^{74}	32	42	18.229	3	3.661	5
Ge^{76}	32	44	17.107	2	3.135	44
As^{75}	33	42	19.595	32	2.417	36
As^{77}	33	44	18.276	11	2.544	27
As^{79}	33	46	17.008	108	2.164	445
Se^{76}	34	42	20.608	9	3.366	12
Se^{78}	34	44	19.224	8	3.318	12
Se^{80}	34	46	18.107	4	3.165	13
Se^{82}	34	48	17.158	6	2.348	108
Br^{79}	35	44	20.379	7	2.607	23
Br^{81}	35	46	19.372	5	2.472	21
Br^{83}	35	48	18.419	22	2.107	24
Kr^{80}	36	44	21.323	6	3.401	29
Kr^{82}	36	46	20.226	6	3.362	214
Kr^{84}	36	48	19.311	4	3.277	8
Kr^{86}	36	50	18.218	4	2.926	13
Kr^{88}	36	52	13.671	235	1.850	257
Rb^{87}	37	50	19.941	5	1.395	15
Rb^{89}	37	52	14.860	54	1.684	200
Sr^{86}	38	48	21.477	5	3.259	66
Sr^{88}	38	50	20.975	6	2.851	9
Sr^{90}	38	52	15.210	32	1.502	37
Sr^{92}	38	54	14.137	87	1.595	93
Y^{87}	39	48	23.365	220	2.515	243
Y^{89}	39	50	22.148	215	2.474	216
Y^{91}	39	52	15.923	13	1.181	65
Y^{93}	39	54	15.068	27	0.954	69
Zr^{92}	40	52	16.999	7	1.537	11
Zr^{94}	40	54	16.041	6	1.543	21
Zr^{96}	40	56	15.380	7	1.488	27
Nb^{91}	41	50	23.624	128	2.628	130
Nb^{93}	41	52	17.750	82	1.198	125
Nb^{95}	41	54	16.874	7	1.420	54
Nb^{97}	41	56	16.075	10	1.183	65
Mo^{92}	42	50	24.505	108	2.377	185
Mo^{94}	42	52	19.052	5	2.012	140
Mo^{96}	42	54	17.748	4	1.916	8
Mo^{98}	42	56	16.599	4	1.959	6
Mo^{100}	42	58	15.267	5	2.125	644

[a] Binding energy of the last pair of neutrons in milliunits.
[b] Pairing energy of the last pair of neutrons in milliunits.

TABLE IX. Binding energy and pairing energy of the last pair of protons for the listed isotopes.

Isotope	Z	N	S_{2p}[a] mu	Error	P_p[b] mu	Error
Ge^{69}	32	37	14.832	12	0.886	25
Ge^{70}	32	38	16.260	9	2.028	11
Ge^{71}	32	39	17.343	29	0.421	50
Ge^{72}	32	40	18.914	16	1.978	18
Ge^{73}	32	41	20.155	220	1.307	220
Ge^{74}	32	42	22.204	220	1.330	235
Se^{73}	34	39	13.785	34		
Se^{74}	34	40	15.241	4	3.159	64
Se^{75}	34	41	16.578	4	1.886	17
Se^{76}	34	42	17.620	8	2.806	10
Se^{77}	34	43	18.587	22	2.013	33
Se^{78}	34	44	19.737	3	2.585	22
Se^{79}	34	45	20.740	55	1.680	433
Kr^{77}	36	41	13.693	22	1.267	34
Kr^{78}	36	42	14.475	9	3.191	15
Kr^{79}	36	43	15.483	15	2.333	26
Kr^{80}	36	44	16.574	5	2.980	7
Kr^{81}	36	45	17.535	107	1.983	109
Kr^{82}	36	46	18.693	4	2.569	9
Kr^{83}	36	47	19.305	54	1.679	55
Kr^{84}	36	48	20.846	5	2.200	44
Sr^{84}	38	46	15.691	4		
Sr^{85}	38	47	16.783	33	1.641	39
Sr^{86}	38	48	17.857	4	2.811	9
Sr^{87}	38	49	19.273	7	0.973	16
Sr^{88}	38	50	20.614	5	2.122	8
Sr^{89}	38	51	21.556	53	1.712	199
Sr^{90}	38	52	22.150	237	1.286	259
Zr^{89}	40	49	15.599	11	1.239	19
Zr^{90}	40	50	16.578	7	1.422	10
Zr^{91}	40	51	17.455	10	1.265	65
Zr^{92}	40	52	18.367	32	1.785	40
Zr^{93}	40	53	19.345	19	1.191	67
Zr^{94}	40	54	20.271	81	1.827	94
Mo^{91}	42	49	12.977	76	2.005	79
Mo^{92}	42	50	13.526	6	2.086	164
Mo^{93}	42	51	14.315	70	1.785	117
Mo^{94}	42	52	15.579	5	2.637	9
Mo^{95}	42	53	16.246	11	2.348	55
Mo^{96}	42	54	17.286	5	2.678	13
Mo^{97}	42	55	17.660	13	2.052	65
Mo^{98}	42	56	18.505	6	2.507	17
Mo^{99}	42	57	19.086	322		

[a] Binding energy of the last pair of protons in milliunits.
[b] Pairing energy of the last pair of protons in milliunits.

nucleus of Z protons and N neutrons is given by Eq. (5).

$$P_n(Z,N)=\mathrm{TNBE}(Z,N)-2\mathrm{TNBE}(Z,N-1)+\mathrm{TNBE}(Z,N-2), \quad (5)$$

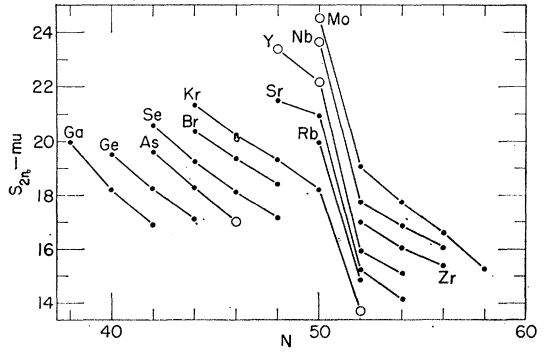

FIG. 6. Binding energy of the last two neutrons.

where N is even. The proton pairing energy P_p is defined in a similar manner. For light nuclei Mayer and Jensen[26] have concluded that a correlation exists be-

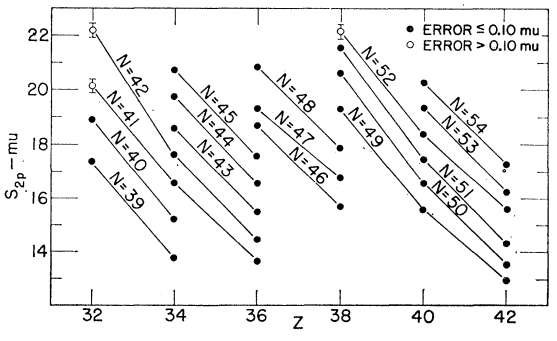

FIG. 7. Binding energies of the last two protons.

[26] M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York, 1955).

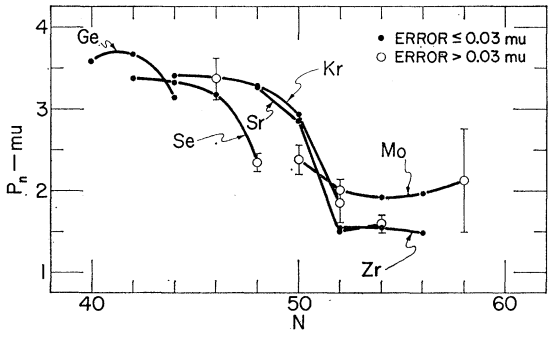

FIG. 8. Neutron pairing energy.

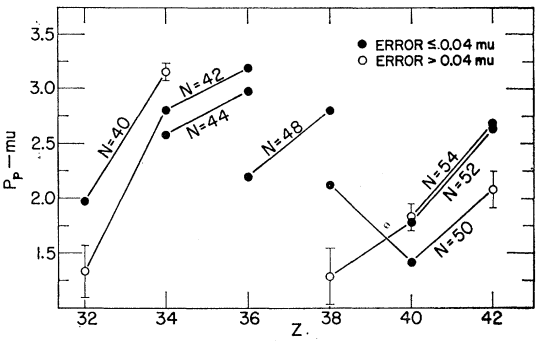

FIG. 9. Proton pairing energy.

tween the pairing energy and the j value of the odd nucleon of the pair, with larger pairing energies correlated to higher j values. In regions of high j values, one finds that it is energetically possible to have the pair occupy a high j-value state rather than to pair in the lower spin state of the preceding odd nucleon. This mechanism is used to explain the absence of the highest j values from the ground-state spins of odd nuclei.

Neutron and proton pairing energies that may be calculated from the present masses are listed in Tables VIII and IX, respectively. The P_n values are plotted in Fig. 8 as a function of N. Values from the same element are connected by a curved line. An attempt has been made to correlate the magnitude of these pairing energies with (a) the j value of the previous odd neutron and (b) with the j value which the pair is assumed to have according to the filling scheme of Mayer and Jensen.[26] Neither comparison produces very convincing results. The correlation between the value of j and the pairing energy is in some cases what Mayer and Jensen have suggested; in others, the opposite. The one positive statement that may be made is that the value of P_n decreases, rather strikingly, following the shell closure at $N=50$. The general tendency of the curves for each element is smooth, and in most instances continuously decreasing with increasing N to $N=50$. The character of the curves changes abruptly following $N=50$. Beyond $N=50$, the P_n values are small, and for each element, are essentially constant for the region plotted. Note that there is nothing anomalous about the one value at $N=40$. Values of P_n for zinc from Quisenberry *et al.*[1] at $N=40$ and $N=38$ further strengthen this conclusion.

Figure 9 is an illustration of the proton pairing energy P_p as a function of Z. In this illustration, points with the same N value are connected by lines. An attempt to correlate the P_p with the j value for the pair is again not particularly fruitful. The P_p values at $Z=40$, with $j=\frac{1}{2}$ for this pair, appear to be lower than practically all other values in this region. A j value of $\frac{1}{2}$ occurs only at $Z=40$; values for other pairs in this region are all larger. Thus, in this instance, low P_p is correlated with low j. There are other instances, however, where this correlation is reversed. It is of interest to note that the value of P_p at $Z=40$ is so small. This may indicate a structure change near $Z=40$ that is not indicated at $N=40$.

ACKNOWLEDGMENTS

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